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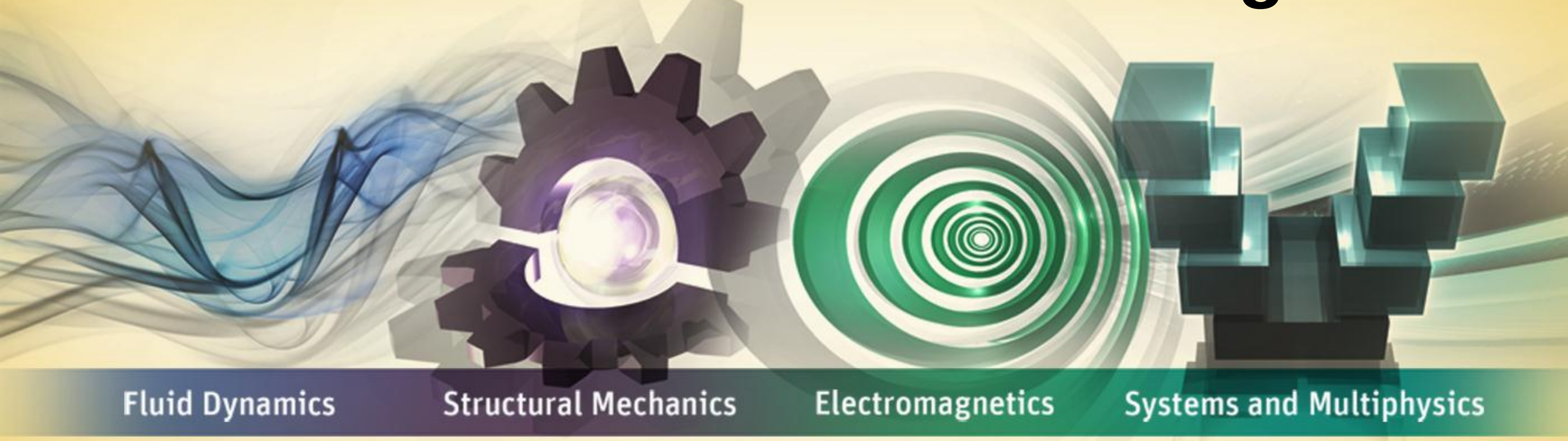
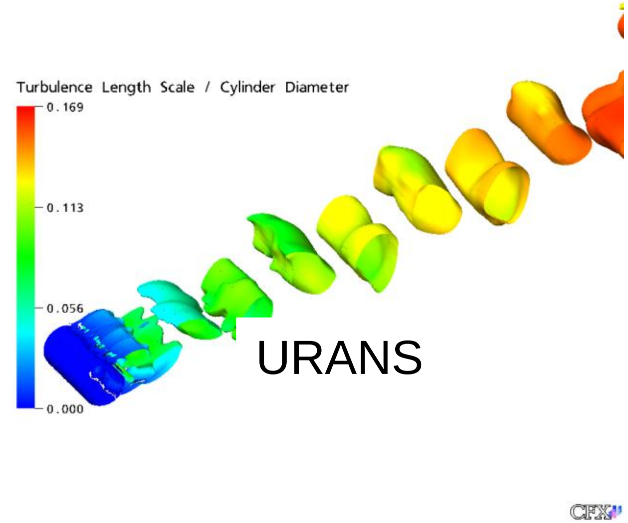
Scale-Adaptive Simulation (SAS) Turbulence Modeling



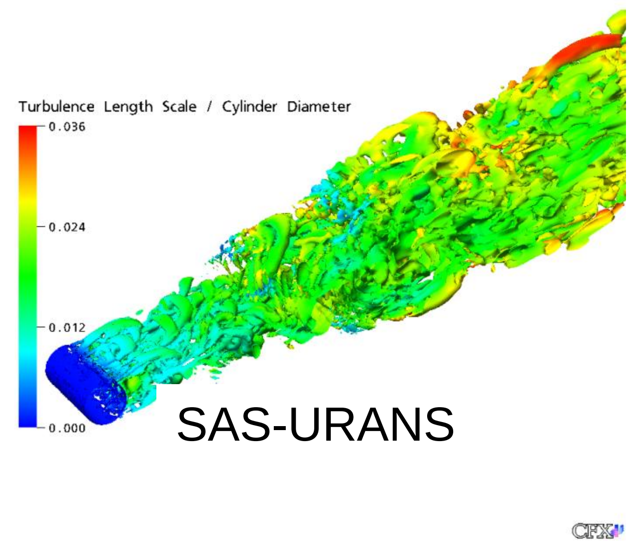
F.R. Menter,
ANSYS Germany GmbH

Unsteady RANS Based Models

- URANS (Unsteady Reynolds averaged Navier Stokes) Methods
  - URANS gives unphysical single mode unsteady behavior
  - Some improvement relative to steady state (RANS) but often not sufficient to capture main effects
  - Reduction of time step and refinement of mesh do not benefit the simulation



- SAS (Scale-Adaptive Simulation) Method
  - Extends URANS to many technical flows
  - Provides “LES”-content in unsteady regions
  - Produces information on turbulent spectrum
  - Can be used as basis for acoustics simulations



Assumptions Two-Equation Models

- Largest eddies are most effective in mixing
- Two scales are minimum for statistical description of large turbulence scales
- Two model equations of independent variables define the two scales
  - Equation for turbulent kinetic energy is representing the large scale turbulent energy
  - Second equation (ε , ω, *kL*) to close the system
  - Each equation defines one independent scale
  - Both ε- and ω-equations describe the smallest (dissipate) eddies, whereas two-equation models describe the largest scales
  - Rotta developed an exact transport equation for the large turbulent length scales. This is a much better basis for a term-by-term modelling approach

Classical Derivation 2 Equation Models

- The k-equation:
  - Can be derived exactly from the Navier-Stokes equations
  - Term-by-term modelling
- The ε- (ω-) equation:
  - Exact equation for smallest (dissipation) scales
  - Model for large scales not based on exact equation
  - Modelled in analogy to k-equation and dimensional analysis
  - Danger that not all effects are included

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \overline{U}_j k)}{\partial x_j}=P_k-c_\mu \rho k\omega+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

$$\frac{\partial(\rho \omega)}{\partial t}+\frac{\partial(\rho \overline{U}_j \omega)}{\partial x_j}=\alpha\left(\frac{\omega}{k}\right)P_k-\beta\left(\frac{\omega}{k}\right)\rho(k\omega)+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\omega}\frac{\partial \omega}{\partial x_j}\right)$$

$$\mu_t=\rho\frac{k}{\omega}$$

Source Terms Equilibrium – k-ω Model

Only one Scale in Sources (S~1/T)

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\mu_t\left(S^2-c_\mu\omega^2\right)+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho U_j\omega)}{\partial x_j}=\rho\left(c_{\omega 1}S^2-c_{\omega 2}\omega^2\right)+\frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\omega}\frac{\partial\omega}{\partial x_j}\right)$$

Input S → **Turbulence Model** → **Output k**, **Output ω**

One input scale – two output scales?

Source terms do not contain information on two independent scales

Determination of *L* in *k*-ω Model

k-equation: $$\frac{\partial(k)}{\partial t}+\frac{\partial(U_k k)}{\partial x_k}=\frac{k}{\omega}(S^2-c_\mu\omega^2)+\frac{\partial}{\partial y}\left[\frac{k}{\omega}\frac{\partial k}{\partial y}\right]$$

- Diffusion term carries information on shear-layer thickness δ
- Turbulent length scale proportional to shear layer thickness
- Finite thickness layer required
- Computed length scale independent of details inside turbulent layer
- No scale-resolution, as L_t always large and dissipative

$$0=\frac{k}{\omega}(S^2-c_\mu\omega^2)+c\frac{1}{\delta}\left[\frac{k}{\omega}\frac{k}{\delta}\right]$$

$\omega \sim S$ from ω-equation

$$0=cS^2+\tilde{c}\frac{k}{\delta^2} \qquad k \sim S^2\delta^2$$

$$L_t \sim \frac{\sqrt{k}}{\omega} \sim \frac{\sqrt{S^2\delta^2}}{S} \sim \delta$$

Rotta's Length Scale Equation

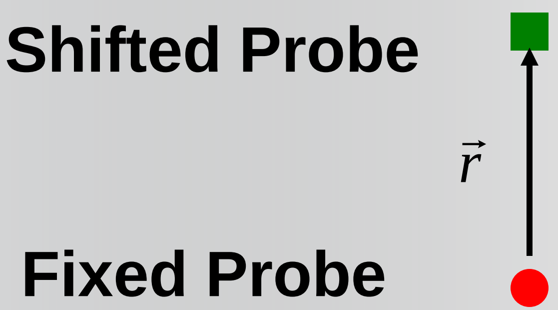
- To avoid the problem that the $\varepsilon\,(\omega)$ equation is an equation for the smallest scales, an equation for the large (integral) scales is needed.
- This requires first a mathematical definition of an integral length scale, *L*.
  - In Rotta's (1968) approach this definition is based on two-point correlations
- Based on that definition of *L,* an exact transport equation can be derived from the Navier-Stokes equations (the actual equation is based on *kL*)
- This exact equation is then modelled term-by-term

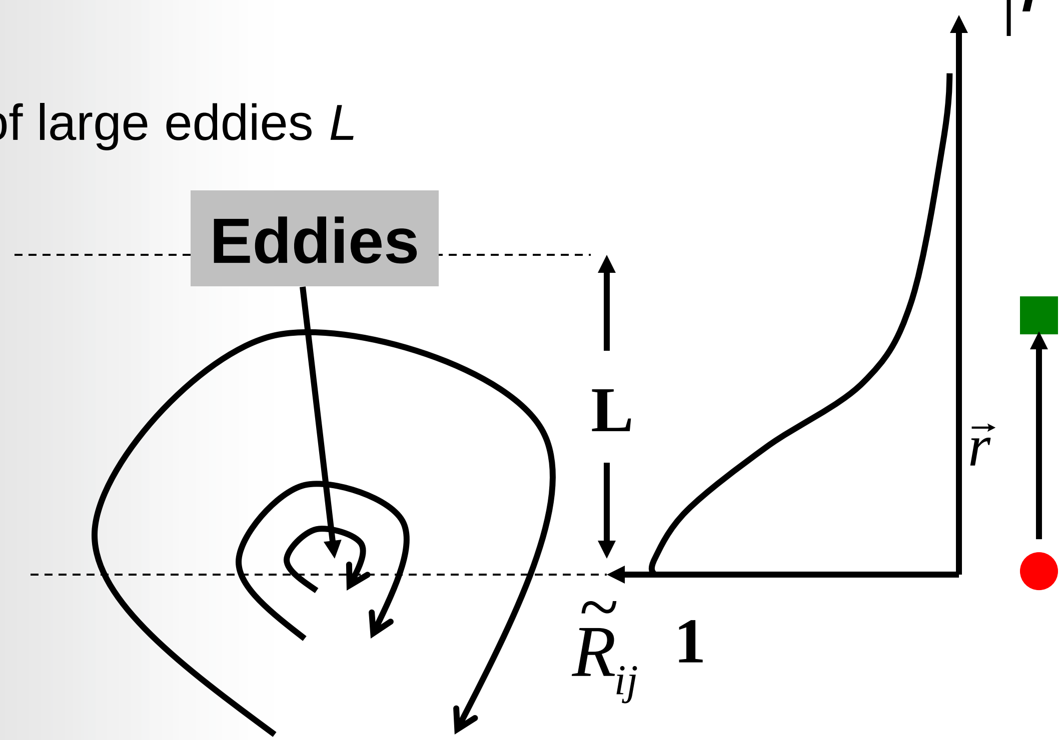
Rotta, J.C.: Über eine Methode zur Berechnung turbulenter Scherströmungen, Aerodynamische Versuchsanstalt Göttingen, Rep. 69 A14, (1968).

Two-Point Velocity Correlations

- Measurement of velocity <u>fluctuations</u> with two probes at two different locations
- For small *r*, all eddies contribute
- For large *r*, only large scales contribute
- For *r* > *L*, correlation goes to zero
- Integral vs. *r* proportional to size of large eddies *L*

$$\tilde{R}_{ij} = \frac{\overline{u_i'(\vec{x},t)\,u_j'(\vec{x}+\vec{r},t)}}{\overline{u_i'(\vec{x},t)\,u_j'(\vec{x},t)}}$$



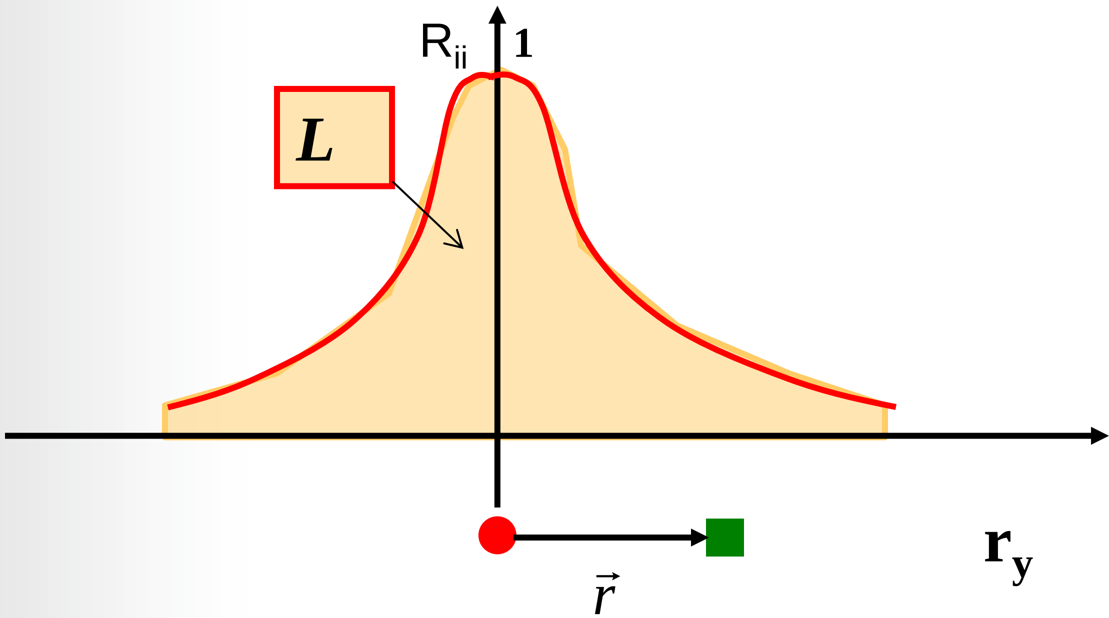


Rotta's *k-kL* Model

Integral Length Scale:

$$L(x,t) = c\int_{-\infty}^{\infty} \tilde{R}_{ii}(x,t,r)dr$$

- The integral of the correlations provides a quantity, *L*, with dimension 'length'.
- *L* is based only on velocity fluctuations and can therefore be described by the Navier-Stokes equations.
- Exact equation for *L* (or *kL*, ..) can be derived.
- *L* is a true measure of the size of the largest eddies



Exact Transport Equation Integral Length-Scale (Rotta)

Exact transport equations for Φ=*kL* (boundary layer form):

$$\frac{\partial(\Phi)}{\partial t}+\frac{\partial(U_k\Phi)}{\partial x_k}=-\frac{3}{16}\frac{\partial U(x)}{\partial y}\int R_{21}dr_y-\frac{3}{16}\int\frac{\partial U(x+r_y)}{\partial y}R_{12}dr_y+$$

$$\frac{3}{16}\int\frac{\partial}{\partial r_k}\left(R_{(ik)i}-R_{i(ik)}\right)dr_y+\nu\frac{3}{8}\int\frac{\partial^2 R_{ii}}{\partial r_k\partial r_k}dr_y-$$

$$\frac{\partial}{\partial y}\left\{\frac{3}{16}\int\left[R_{(i2)i}+\frac{1}{\rho}\left(\overline{p'v}+\overline{vp'}\right)\right]-\nu\frac{\partial}{\partial y}(\Phi)\right\}$$

with $\Phi=kL(x)$

- Important term:

$$\frac{3}{16}\int\frac{\partial U_i(x+r_y)}{\partial y}R_{12}dr_y$$

Expansion of Gradient Function

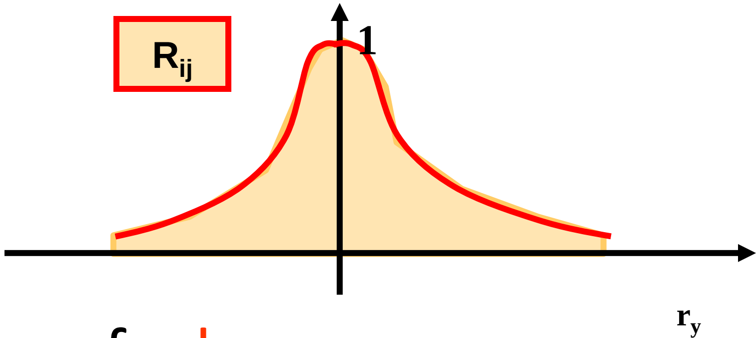
Important term:

$$\frac{\partial U(x+r_y)}{\partial y} = \frac{\partial U(x)}{\partial y} + \frac{\partial^2 U(x)}{\partial y^2} r_y + \frac{\partial^3 U(x)}{\partial y^3}\frac{r_y^2}{2} + \ldots$$

$$\int \frac{\partial U(x+r_y)}{\partial y} R_{12} dr_y \rightarrow \frac{\partial U(x)}{\partial y}\int R_{12} dr_y + \frac{\partial^2 U(x)}{\partial y^2}\int r_y R_{12} dr_y + \frac{1}{2}\frac{\partial^3 U(x)}{\partial y^3}\int r_y^2 R_{12} dr_y$$

- Rotta:

$$\frac{\partial^2 U(x)}{\partial y^2}\int r_y R_{12} dr_y = 0$$



- Due to symmetry of R_{ij} with respect to r_y for homogeneous turbulence

Transport Equation Integral Length-Scale (Rotta)

Transport equations for *kL*:

$$\frac{\partial(\rho\Phi)}{\partial t}+\frac{\partial(\rho U_k\Phi)}{\partial x_k}=-\rho\overline{uv}\left(\zeta L\frac{\partial U_i(x)}{\partial y}+\zeta_3 L^3\frac{\partial^3 U_i(x)}{\partial y^3}\right)-c_L c\rho\left(\frac{q^2}{2}\right)^{3/2}+\frac{\partial}{\partial y}\left\{\frac{\mu_t}{\sigma_\Phi}\frac{\partial}{\partial y}(\Phi)\right\}$$

- Equation has a natural length scale:

$$L^2=\frac{c_l-c}{\zeta_3}\left|\frac{\partial U/\partial y}{\partial^3 U/\partial y^3}\right|$$

- Problem – 3rd derivative:
  - Non-intuitive
  - Numerically problematic

$\longrightarrow$ $$\zeta_3=0$$ $\longrightarrow$

- If ζ_3=0 - No natural length scale
  - No fundamental difference to other scale-equations

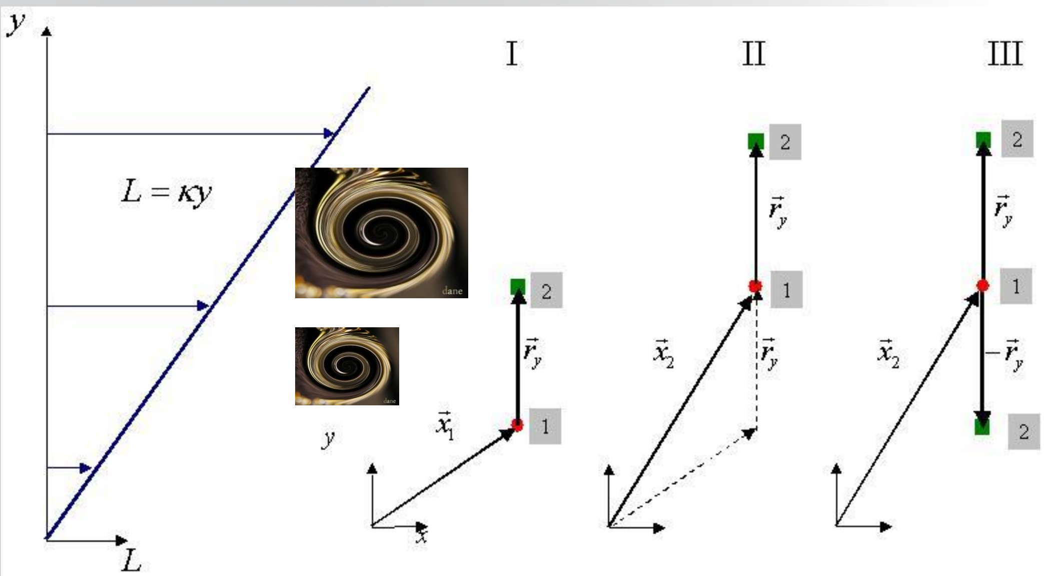
Virtual Experiment 1D Flow

$$\frac{\partial^2 U}{\partial y^2}\int r_y R_{12} dr_y = 0 \ \ ?$$

$$\tilde{R}_{12} = \frac{\overline{u(x)v(x+r_y)}}{\overline{u(x)v(x)}}$$

$$\overline{u(x)v(x)} = const. = \frac{\tau_w}{\rho}$$

Logarithmic layer $L_t=\kappa y$



$$\tilde{R}^{I}_{12}(\vec{r}_y) < \tilde{R}^{II}_{12}(\vec{r}_y)$$

$$\tilde{R}^{III}_{12}(\vec{r}_y) = \tilde{R}^{II}_{12}(\vec{r}_y) \qquad \tilde{R}^{III}_{12}(\vec{r}_y) \approx \tilde{R}^{I}_{12}(-\vec{r}_y)$$

$$\tilde{R}^{III}_{12}(-\vec{r}_y) < \tilde{R}^{III}_{12}(\vec{r}_y)$$

$\longrightarrow$

R_{12} asymmetric

$$\int r_y R_{12} dr_y \neq 0$$

New 2-Equation Model (KSKL)

$$\frac{\partial(k)}{\partial t}+\frac{\partial(U_j k)}{\partial x_j}=P_k-c_\mu^{3/4}\frac{k^{3/2}}{L}+\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

$$\frac{\partial\Phi}{\partial t}+\frac{\partial(U_j\Phi)}{\partial x_j}=\frac{\Phi}{k}\left(\zeta_1 P_k-\zeta_2\frac{1}{\kappa^2}L^2\nu_t\left(U''\right)^2\right)-\zeta_3\cdot k+\frac{\partial}{\partial y}\left[\frac{\nu_t}{\sigma_\Phi}\frac{\partial\Phi}{\partial y}\right]$$

- With:

$$\Phi=\sqrt{k}L \qquad \nu_t=c_\mu^{1/4}\Phi \qquad |U'|=\sqrt{\frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j}}; \quad |U''|=\sqrt{\frac{\partial^2 U_i}{\partial x_j\partial x_j}\frac{\partial^2 U_i}{\partial x_k\partial x_k}}; \quad L_{vK}=\kappa\left|\frac{U'}{U''}\right|$$

v. Karman length-scale as natural length-scale:

$$L\sim\kappa\left|\frac{\partial U/\partial y}{\partial^2 U/\partial y^2}\right|=L_{vK}$$

SAS Model Derivation

- Using the exact definition and transport equation of Rotta, we re-formulated the equation for the second turbulence scale.
- We use a term-by-term modelling approach based on the exact equation.
- This results in the inclusion of the second velocity derivative U'' in the scale equation
- Based on U'' the scale equation is able to adjust to resolved scales in the flow.
- The KSKL model is one variant of the SAS modelling concept, as these terms can also be transformed into other equations (ε- or ω).

Transformation of SAS Terms to SST Model

- Tranformation:

$$\Phi = \frac{1}{c_\mu^{1/4}} \frac{k}{\omega}$$

$$\frac{D\omega}{Dt} = \frac{1}{c_\mu^{1/4}} \frac{D}{Dt}\left(\frac{k}{\Phi}\right) = \frac{1}{c_\mu^{1/4}}\left(\frac{1}{\Phi}\frac{Dk}{Dt} - \frac{k}{\Phi^2}\frac{D\Phi}{Dt}\right) = \frac{\omega}{k}\frac{Dk}{Dt} - \frac{\omega}{\Phi}\frac{D\Phi}{Dt}$$

$$\frac{\partial \rho\omega}{\partial t} + \frac{\partial U_j \rho\omega}{\partial x_j} = \alpha\rho S^2 - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\omega}\frac{\partial \omega}{\partial x_j}\right) + \frac{2\rho}{\sigma_\Phi}\left(\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} - \frac{k}{\omega^2}\frac{\partial \omega}{\partial x_j}\frac{\partial \omega}{\partial x_j}\right) + \tilde{\zeta}_2 \kappa\rho S^2 \left(\frac{L}{L_{vK}}\right)^2$$

Wilcox Model

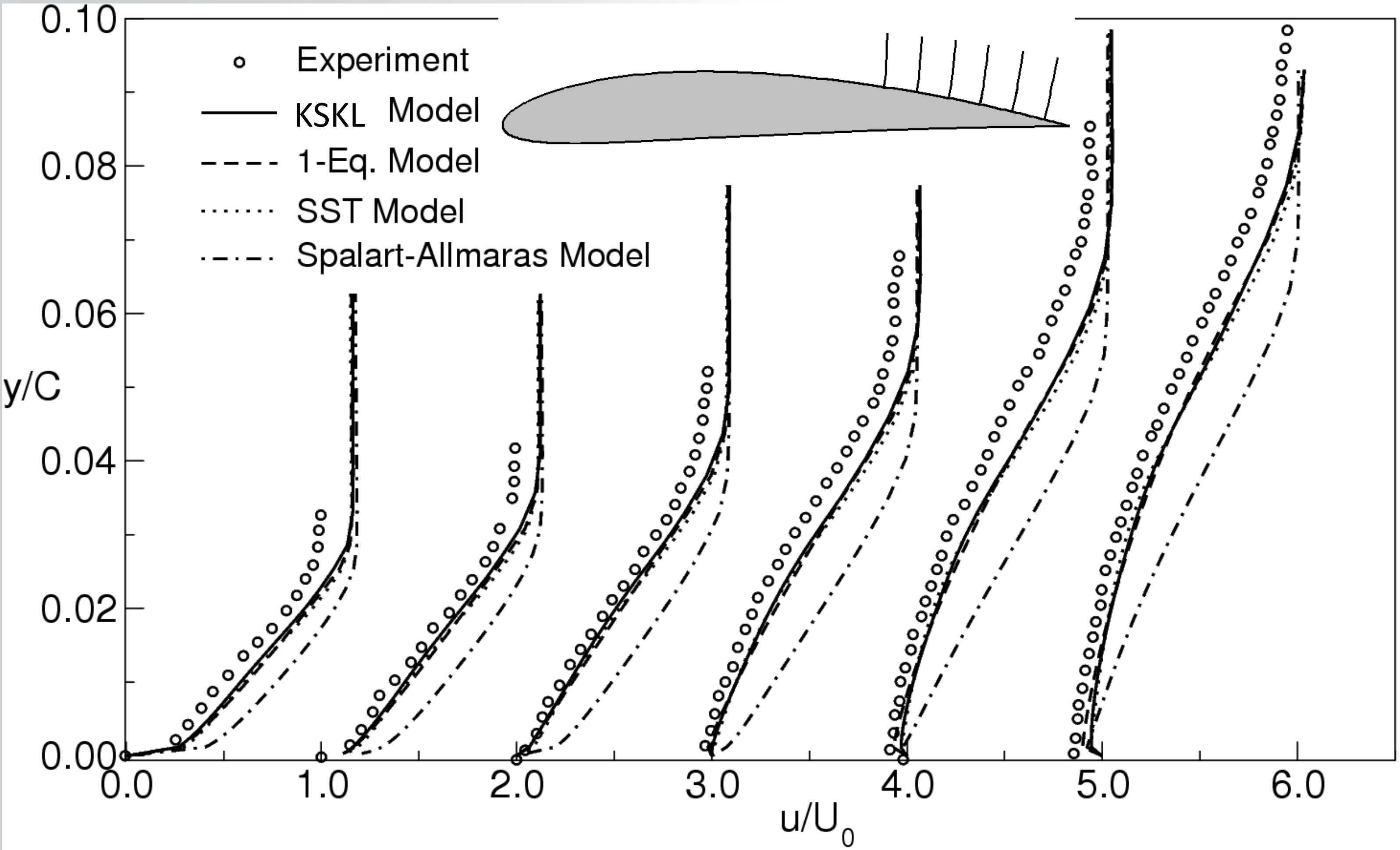
BSL (SST) Model

SAS

$$L_{vK} = \kappa \left| \frac{\partial U / \partial y}{\partial^2 U / \partial y^2} \right|$$

2-D Stationary Flows: KSKL - RANS

NACA-4412 airfoil at 14°: trailing edge separation



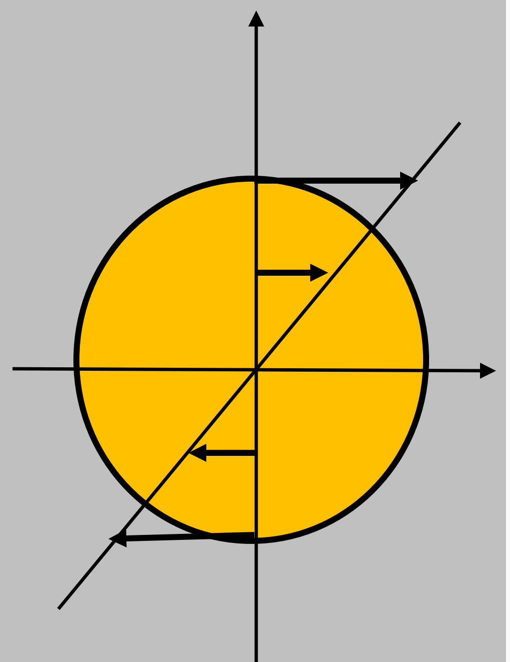
Limitation of Growth by U’’

Homogenous Shear

$$\frac{dU}{dy} = const.$$

$$\omega \sim \frac{dU}{dy}$$

$$L \rightarrow \infty$$



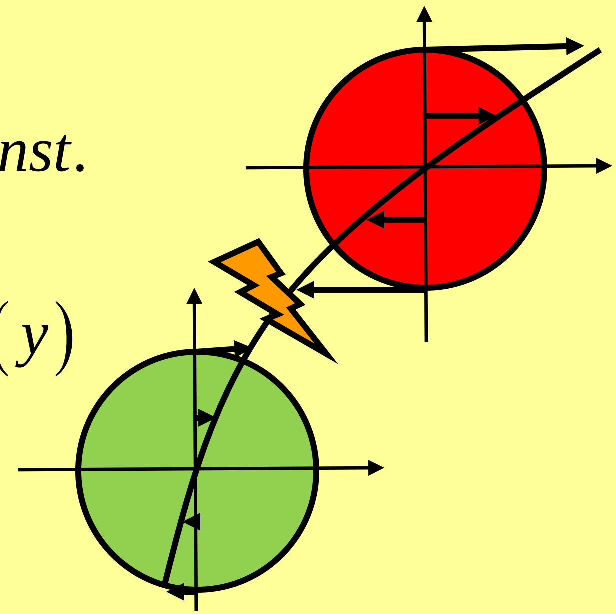
Eddies grow to infinity

Inhomogeneous Shear

$$\frac{dU}{dy} \neq const.$$

$$\omega \sim \frac{dU}{dy}(y)$$

$$L \rightarrow L_{vK}$$



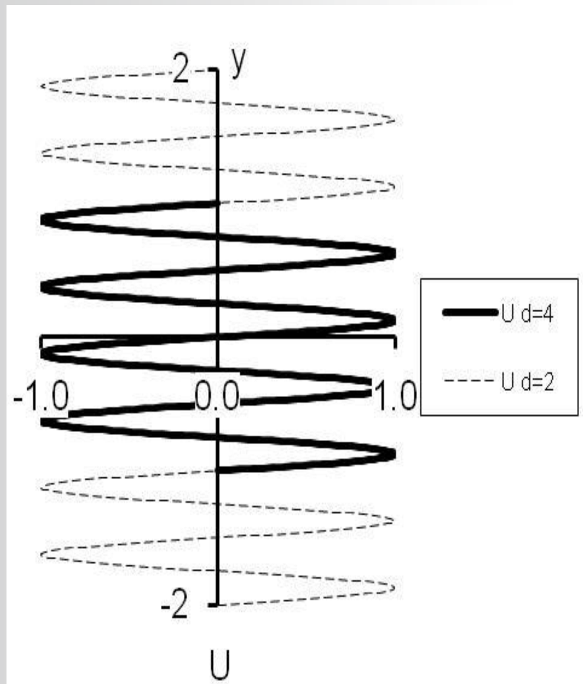
Eddy growth limited by LvK.

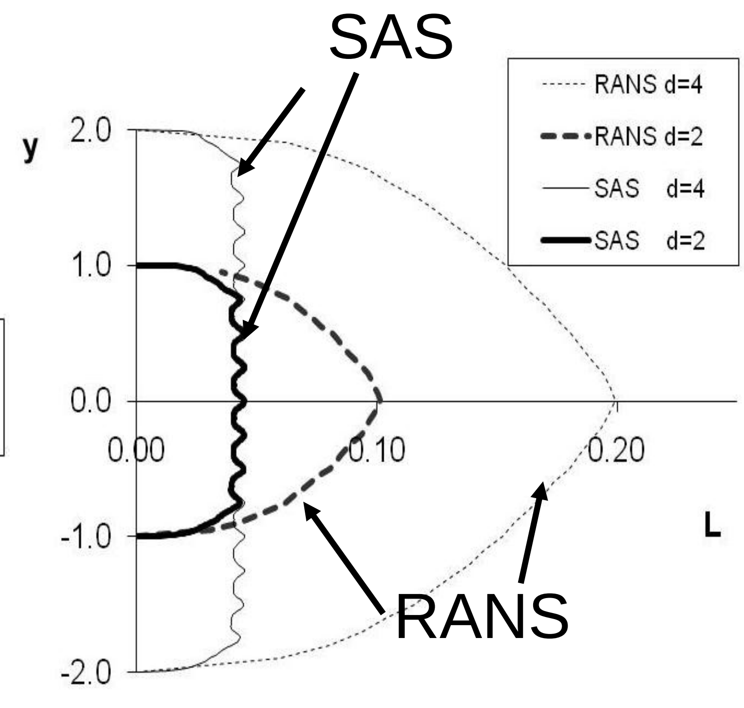
One Model – Two Modes

RANS Model L~δ

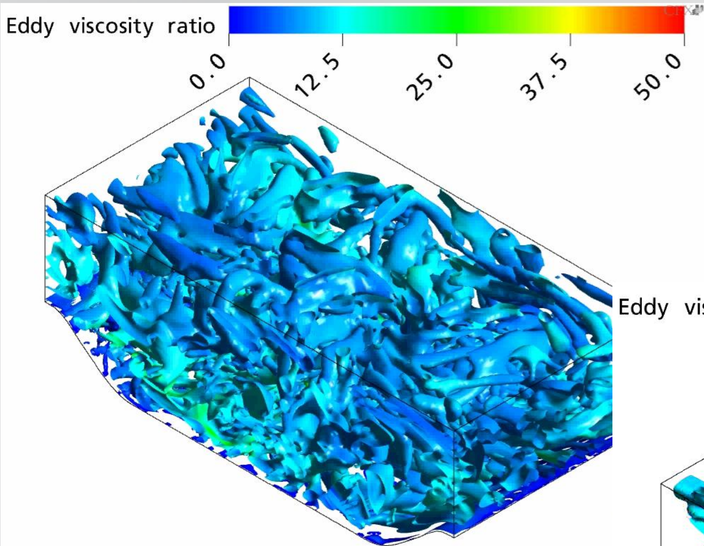
SAS L~λ

$$U(y) = U_0 \sin\left(\frac{2\pi \cdot y}{\lambda}\right)$$



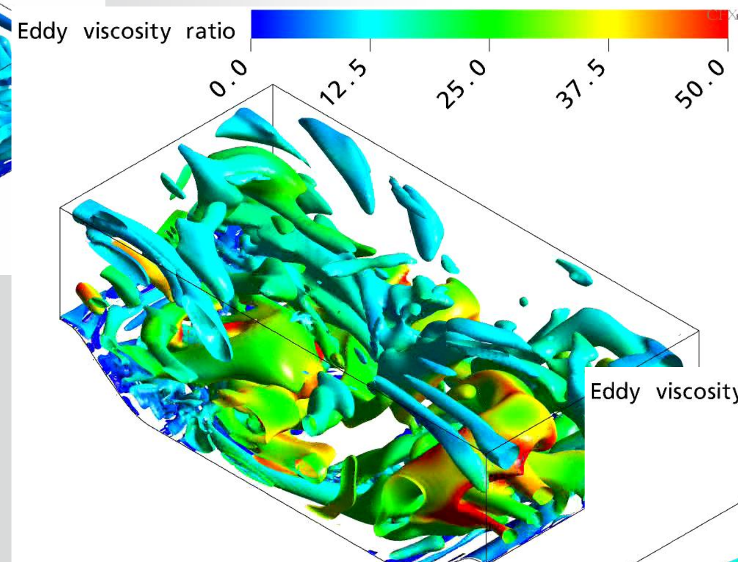


SAS Modell - 2D Periodic Hill

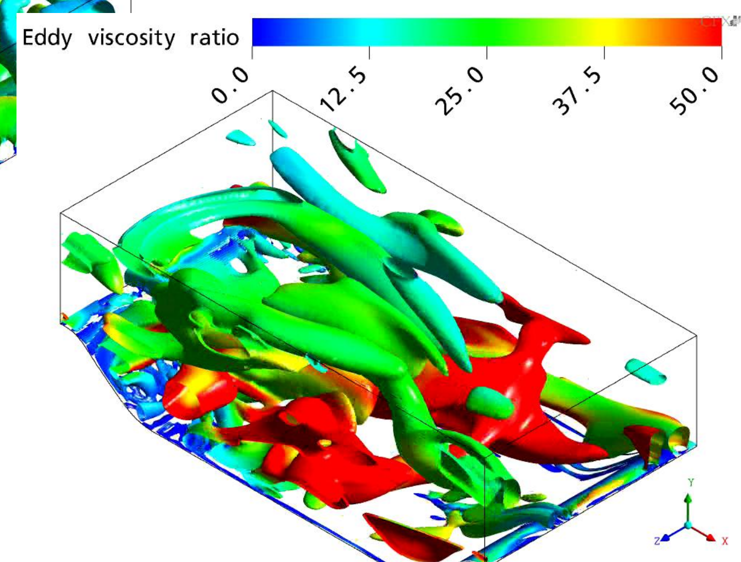


Scale-Adaptation based on Δt

$\Delta t = 0.045\ h/U_B$



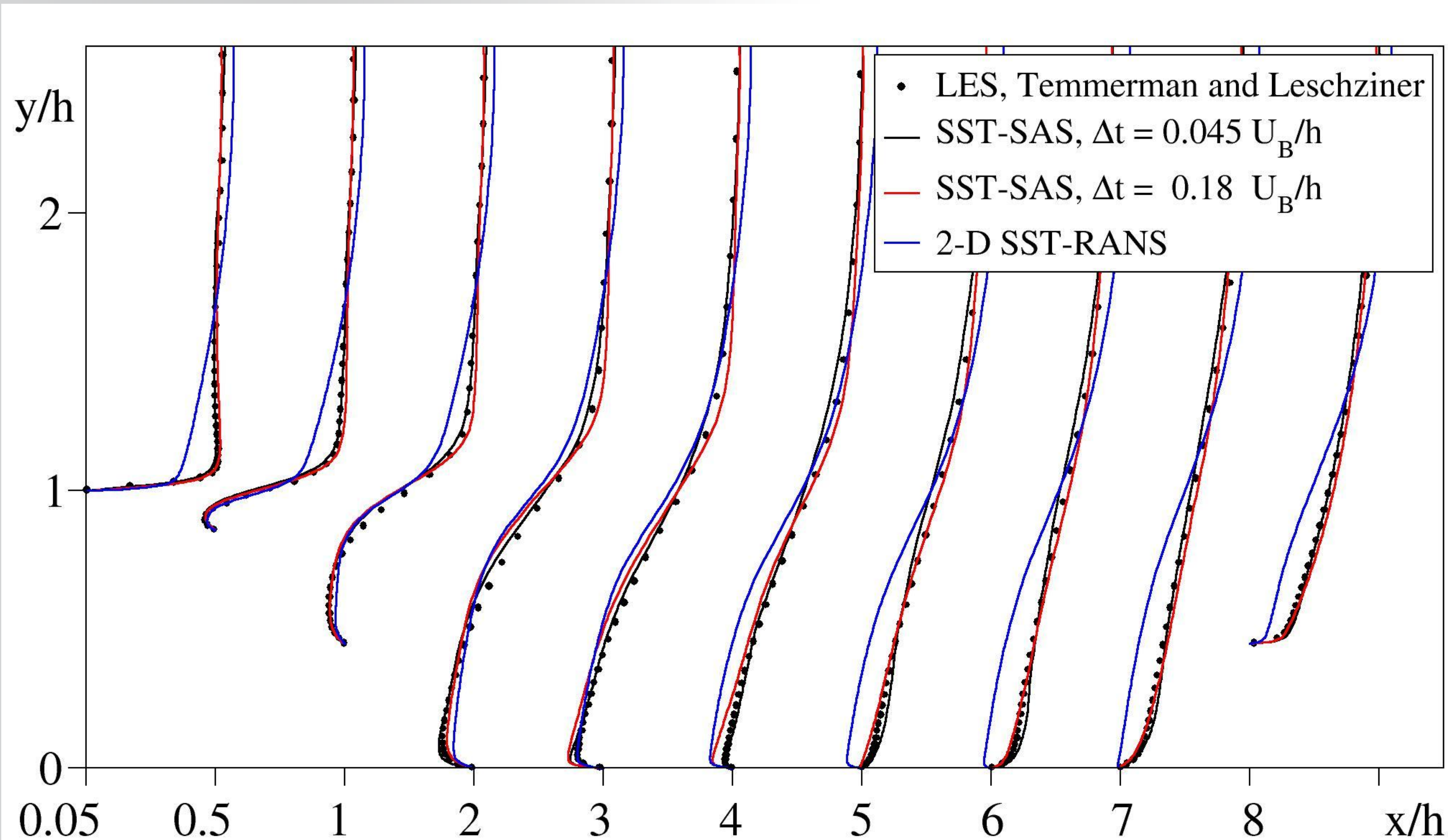
2× higher Δt



4× higher Δt

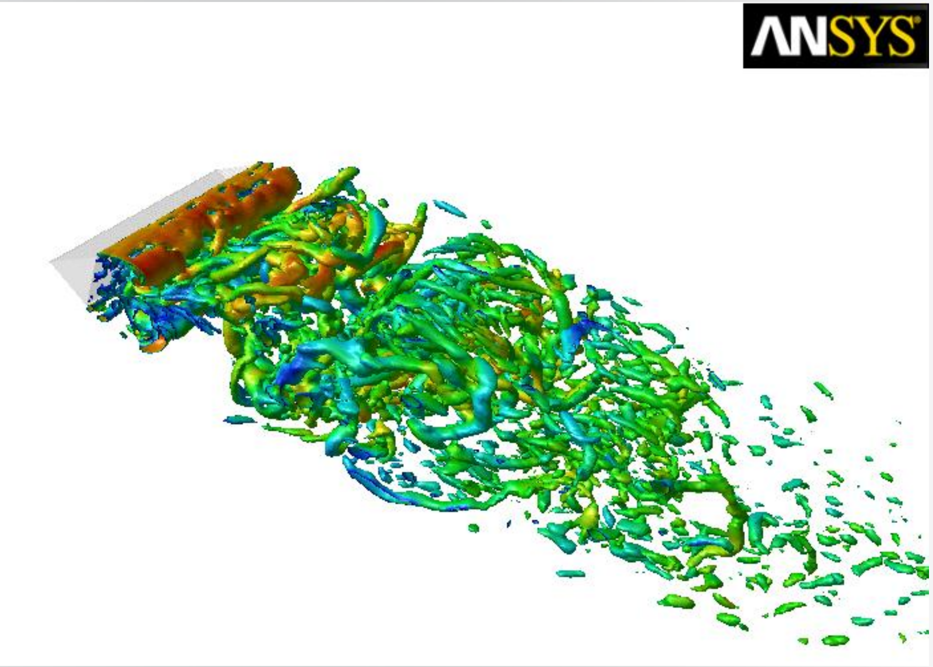
SAS Modell - 2D Periodic Hill

Time averaged velocity profiles U

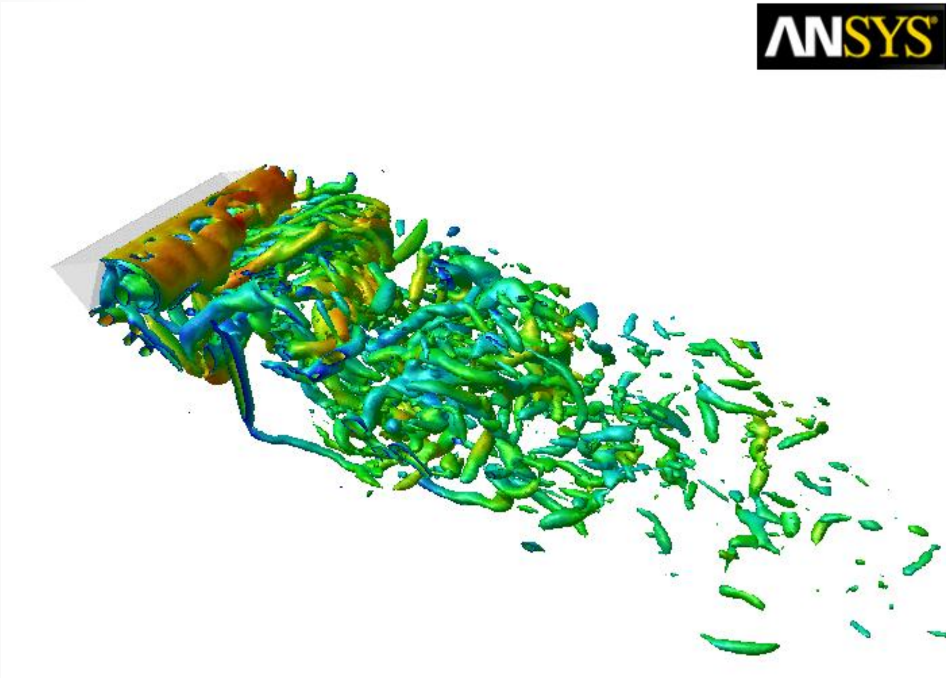


Fluent-SAS Model
Volvo Bluff Body : Cold Case

SAS-SST

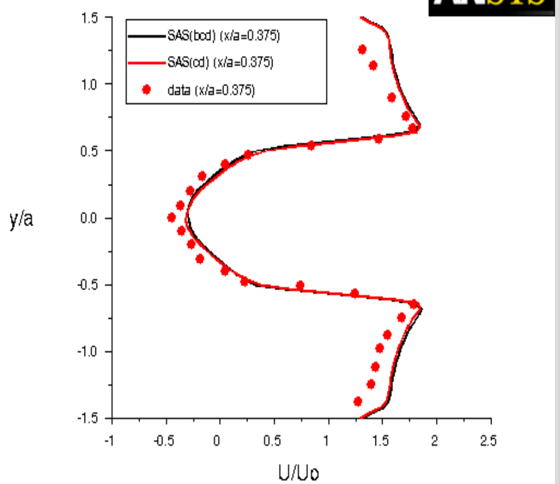


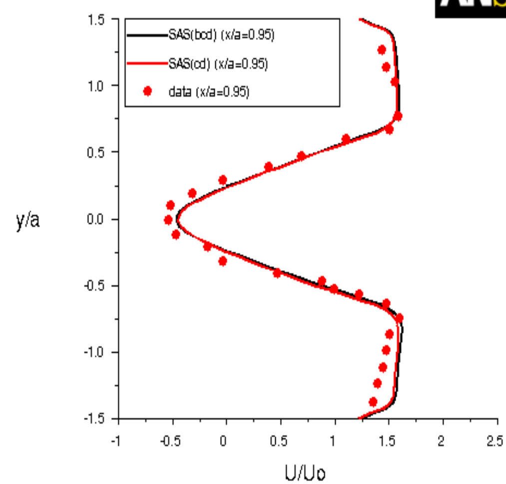
DES-SST

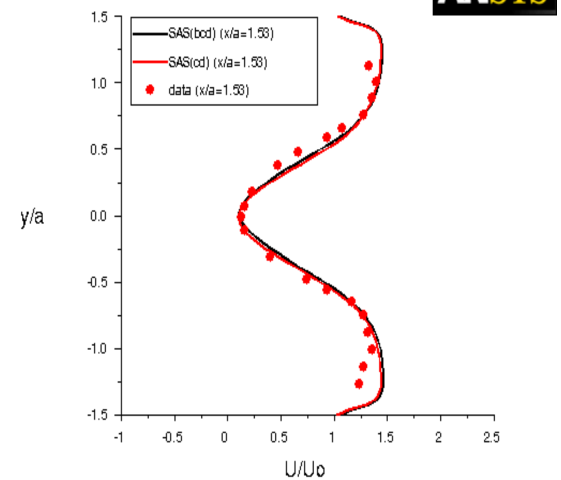


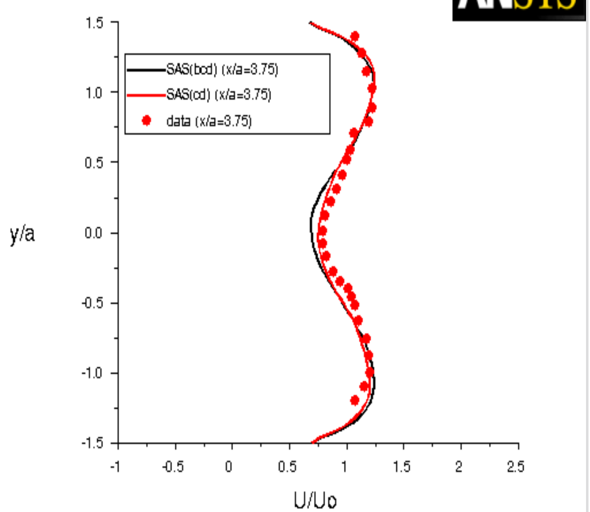
Q = 1^e6

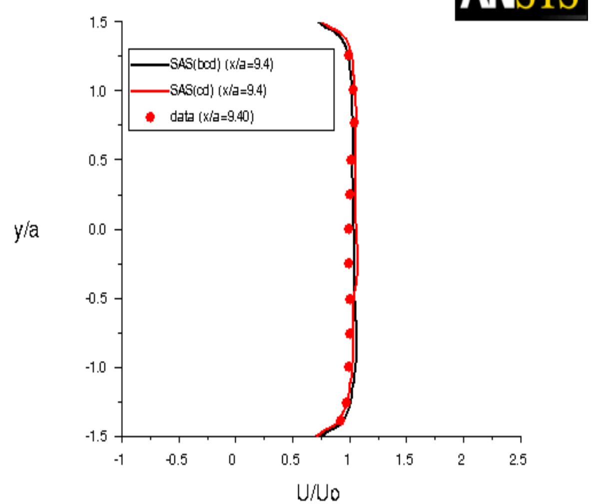
VOLVO Cold Case











Time-averaged U-velocity

Test case: Mirror Geometry

EU project DESIDER Testcase

Plate dimensions L×W= 2.4×1.6

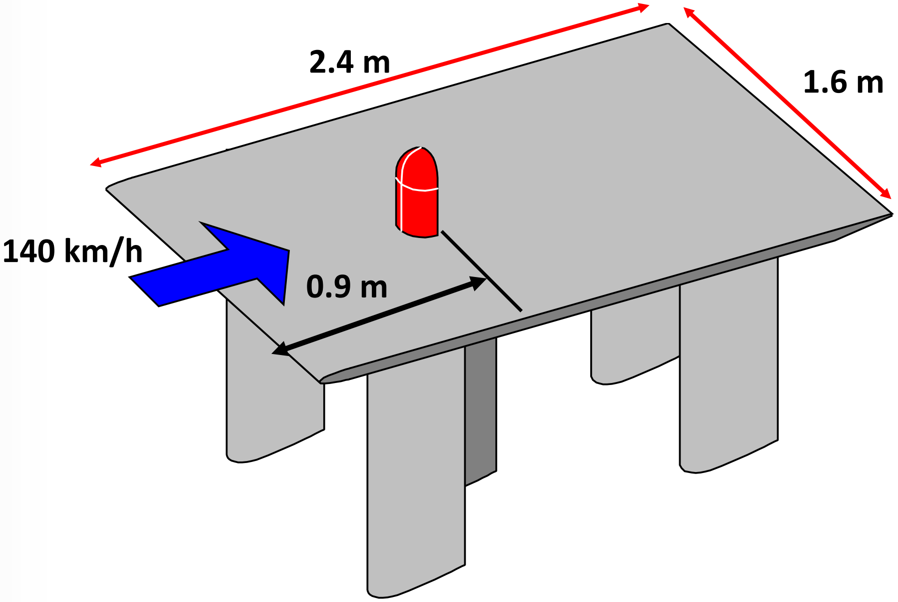
Cylinder Diameter : D = 0.2 m

Rear Face location : 0.9 m

Free stream Velocity: 140 km/h

Re_D: 520 000

Mach: 0.11

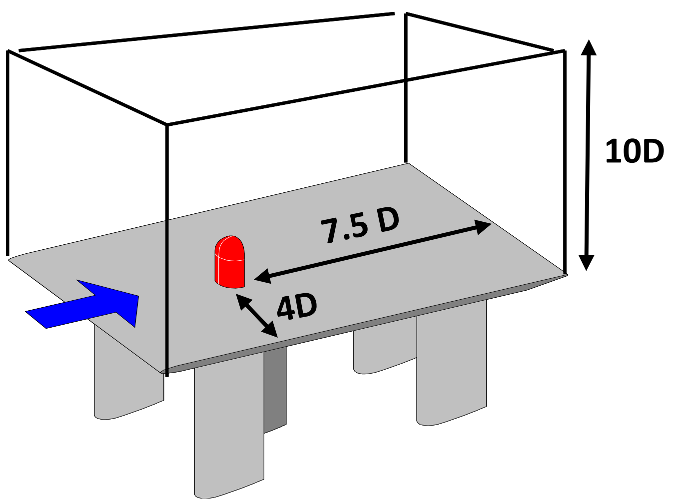


Test case: Mesh

Mesh: Box around the Plate & Cylinder

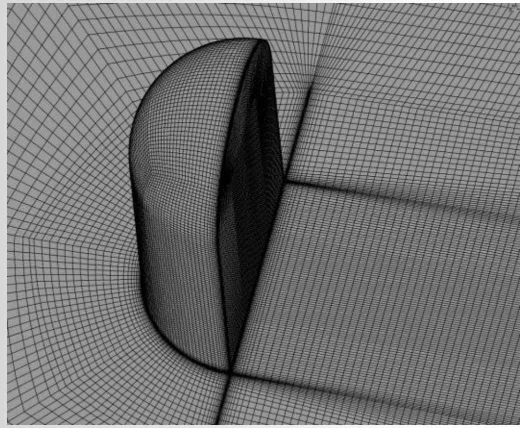
- Height of domain: 10 diameters (D=0.2m)
- Coarse and fine meshes
- wall-normal distance around 1-3 *10 -4 m
- obstacle edges resolution: step sizes around 0.02*D (height) - 0.03*D (circumf.)

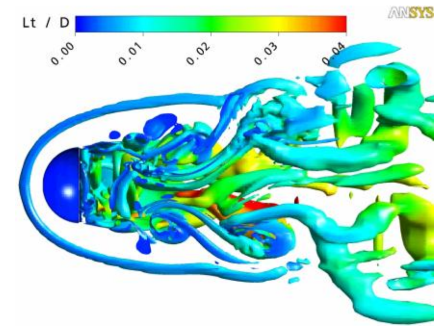
Flow: Air as ideal gas

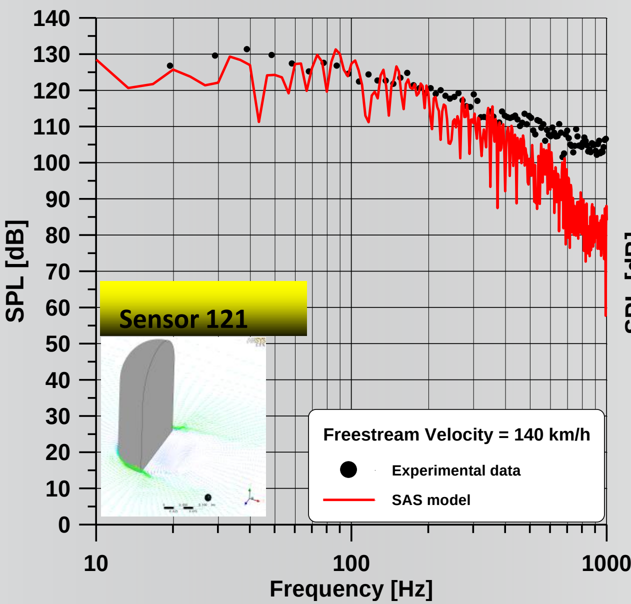


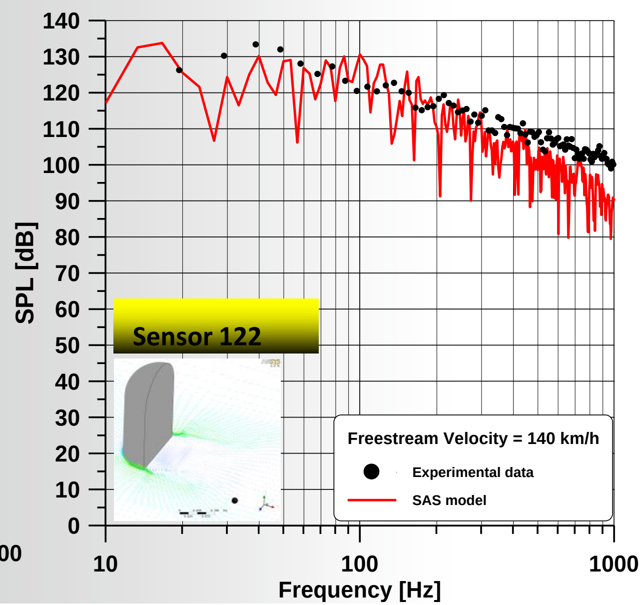
Validation: Near field SPL

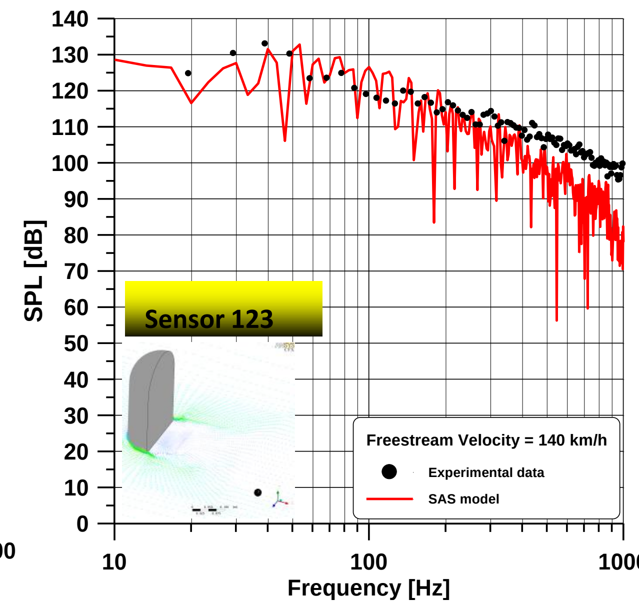
Grid ~ 3 million nodes











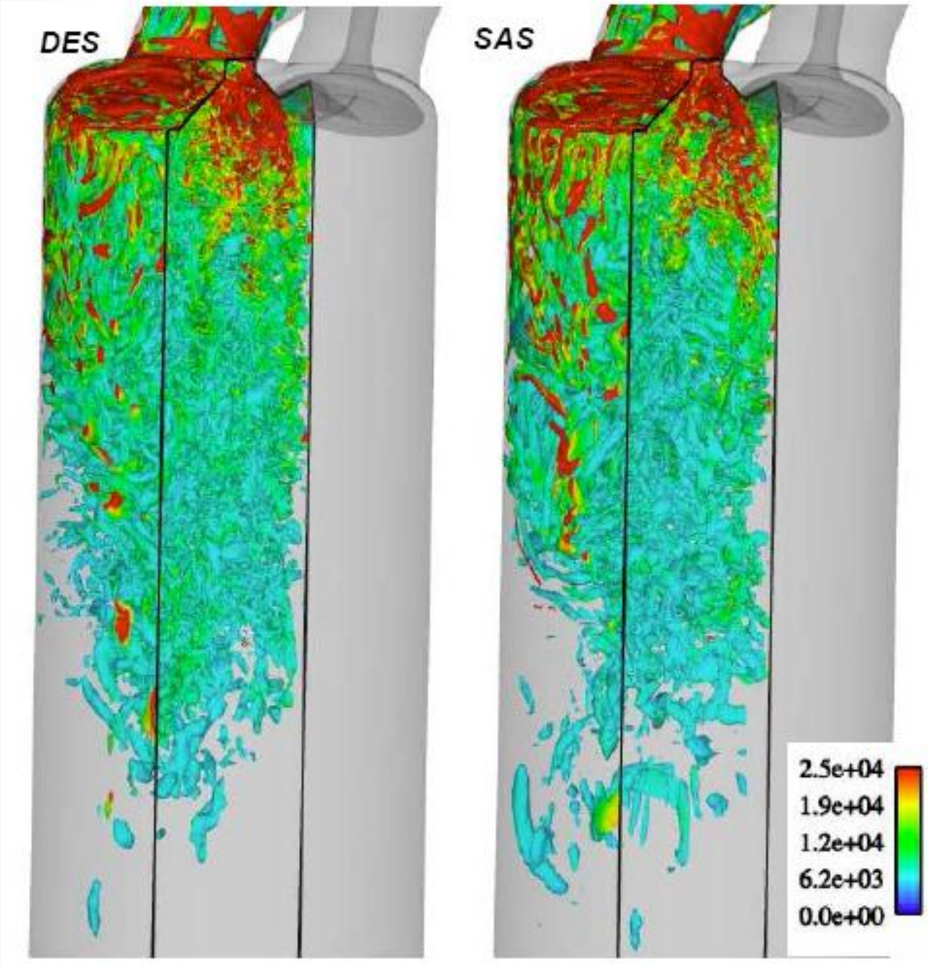
Sensors downstream the mirror

Blow-Down Simulation – SAS (SST)

Mesh – 1×10^7 control volumes hybrid unstructured

Scale resolving results:

- SAS and DES show similar flow pattern
- SAS model does not rely on grid spacing
- SAS can be applied to moving meshes with more confidence

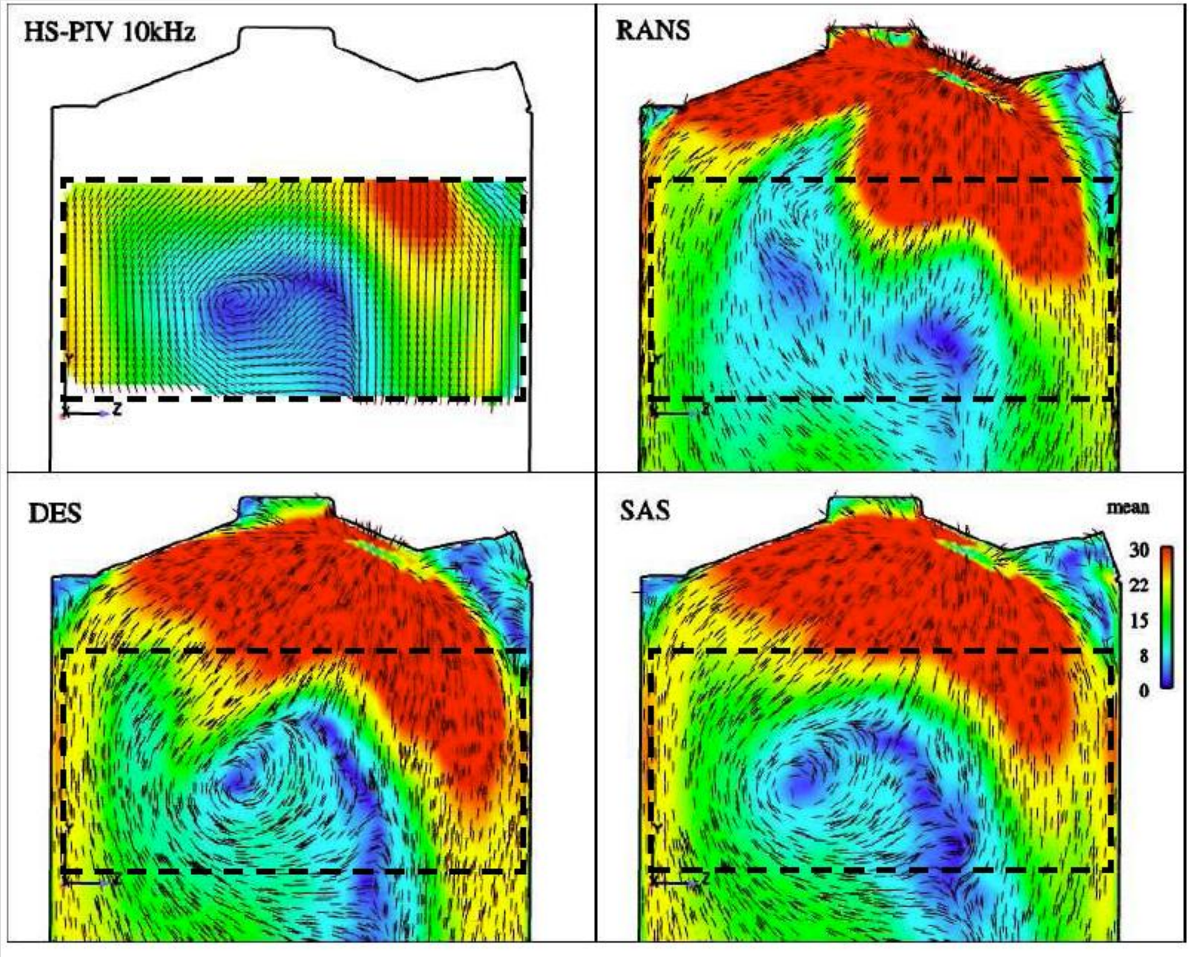


Courtesy VW AG Wolfsburg: O. Imberdis, M. Hartmann, H. Bensler, L. Kapitza
VOLKSWAGEN AG, Research and Development, Wolfsburg, Germany
D. Thevenin University of Magdeburg

Flow Topology and Mass Flow

Mass flow Rates

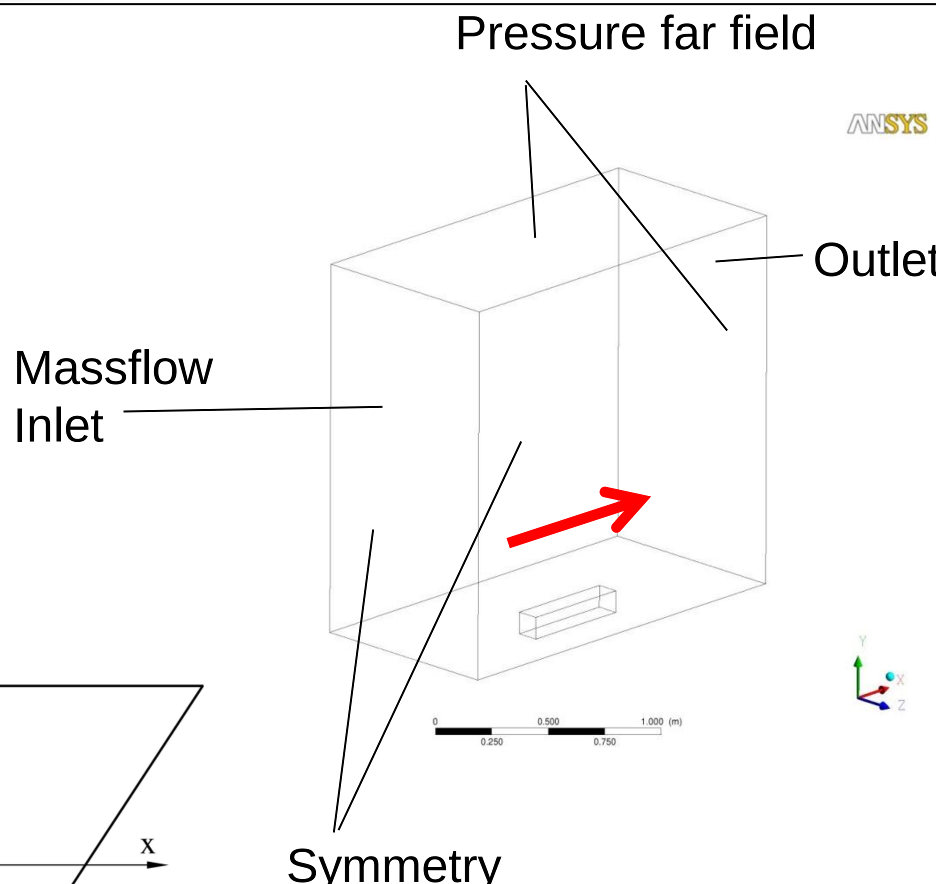
Intake Valve	Exp.	RANS	DES	SAS
3 mm	1	0.95	0.985	0.996
9 mm	1	0.988	-	0.99

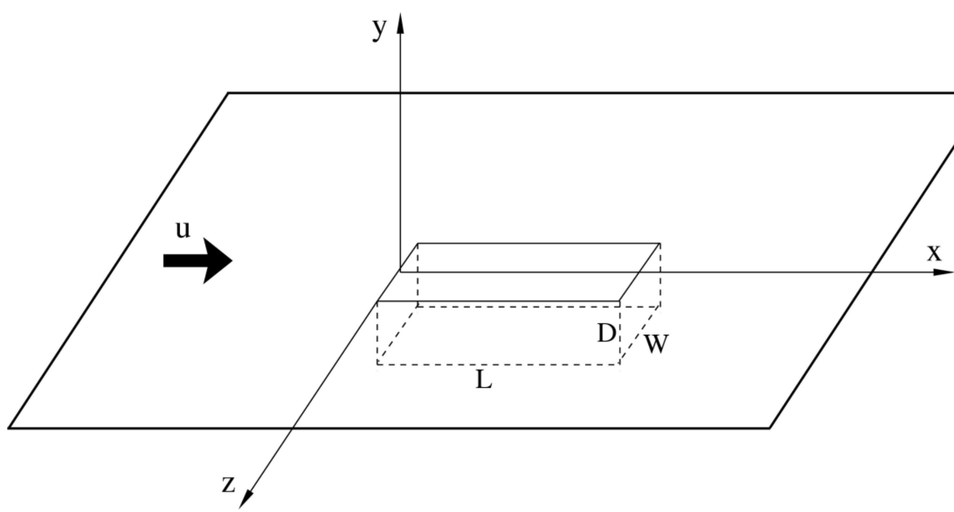


Courtesy VW AG Wolfsburg: O. Imberdis, M. Hartmann, H. Bensler, L. Kapitza
VOLKSWAGEN AG, Research and Development, Wolfsburg, Germany
D. Thevenin University of Magdeburg

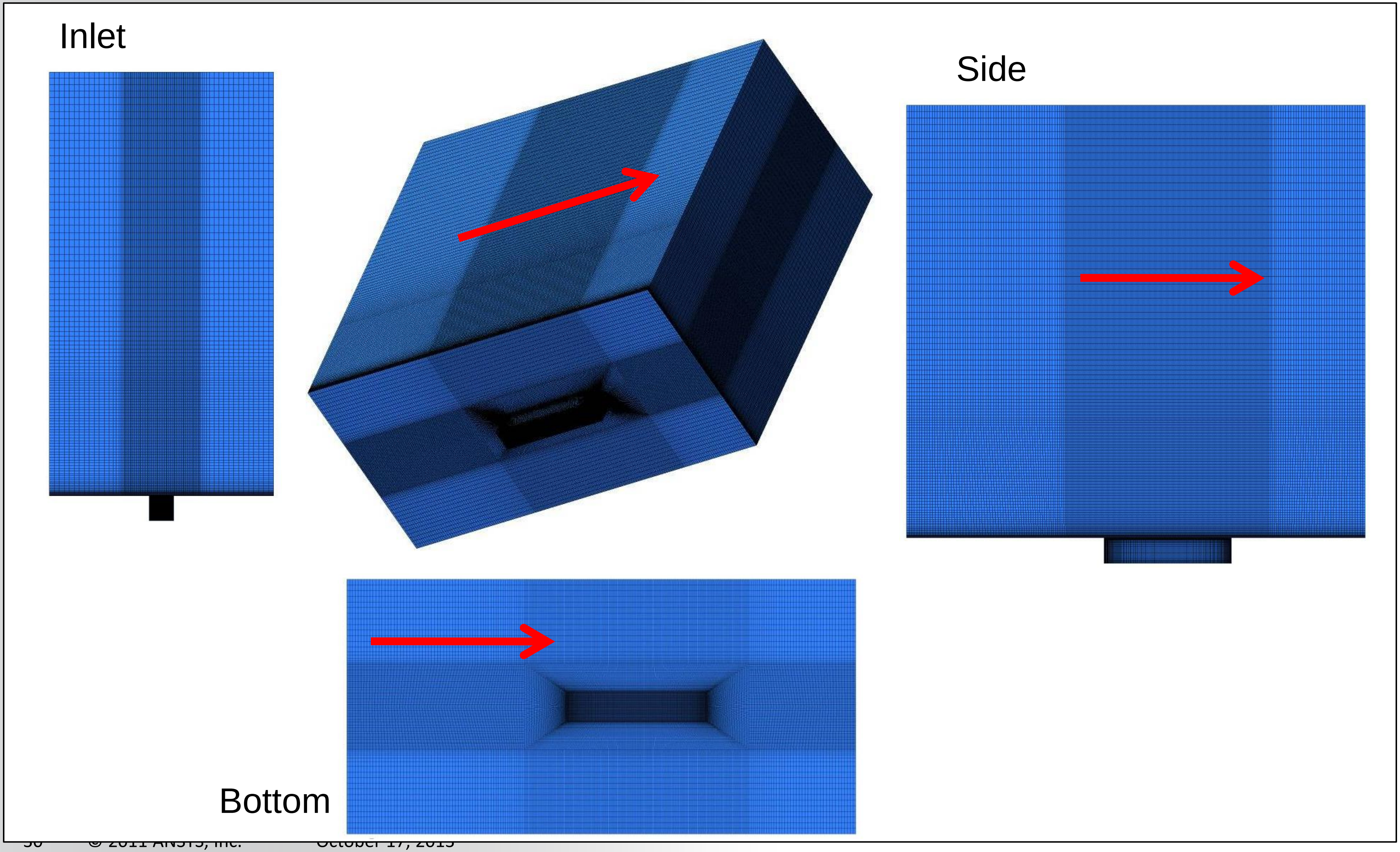
Geometry of the Cavity

- D = 4 in
- L = 5 D, W = D
- $L_x \times L_y \times L_z$ = 18 D x 17 D x 9 D
- M = 0.85
- P = 62100 Pa
- T = 266.53 K
- Re = 13.47 E 6



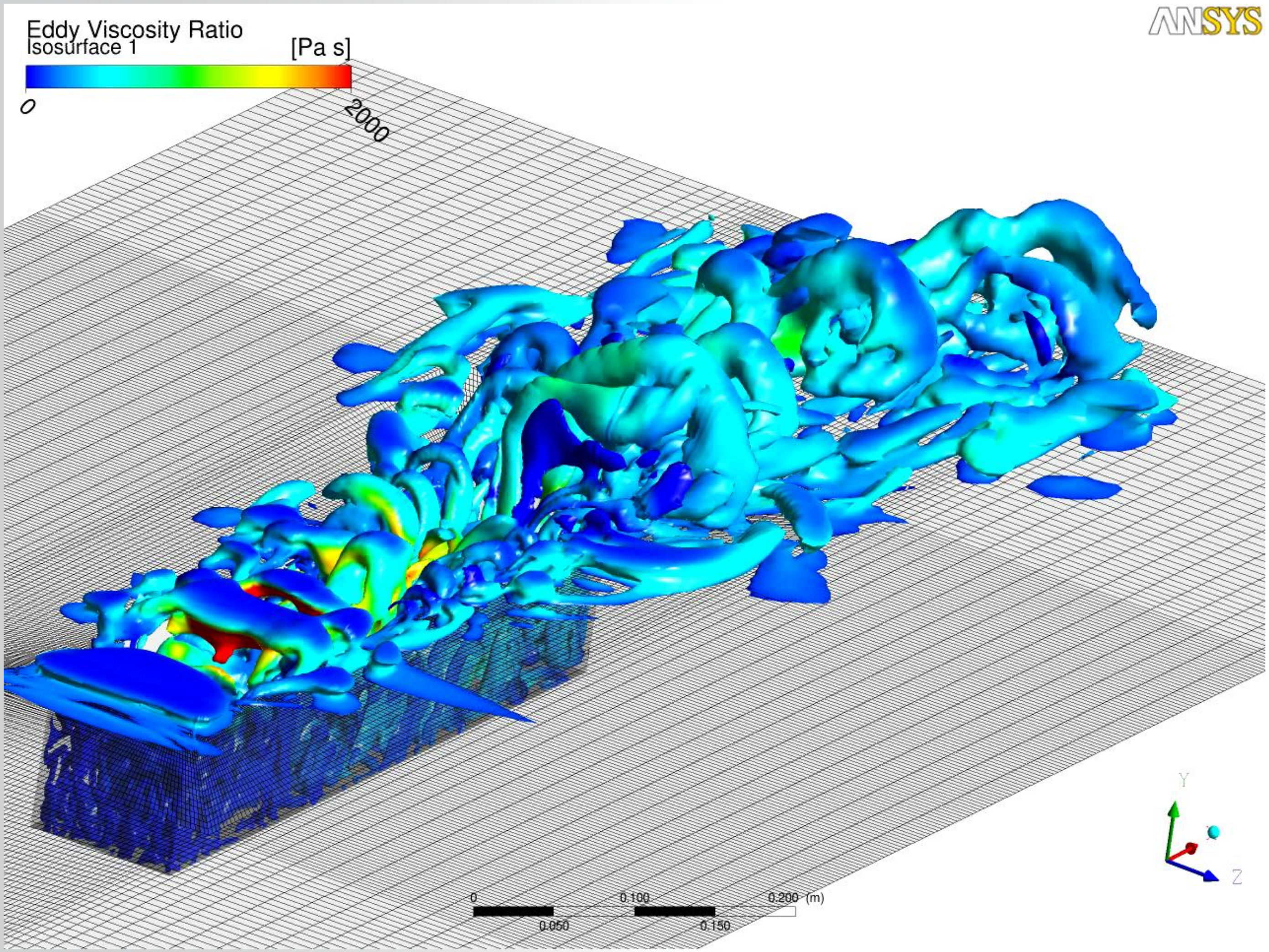


Mesh: 5.8 e 6 Cv – double O-grid



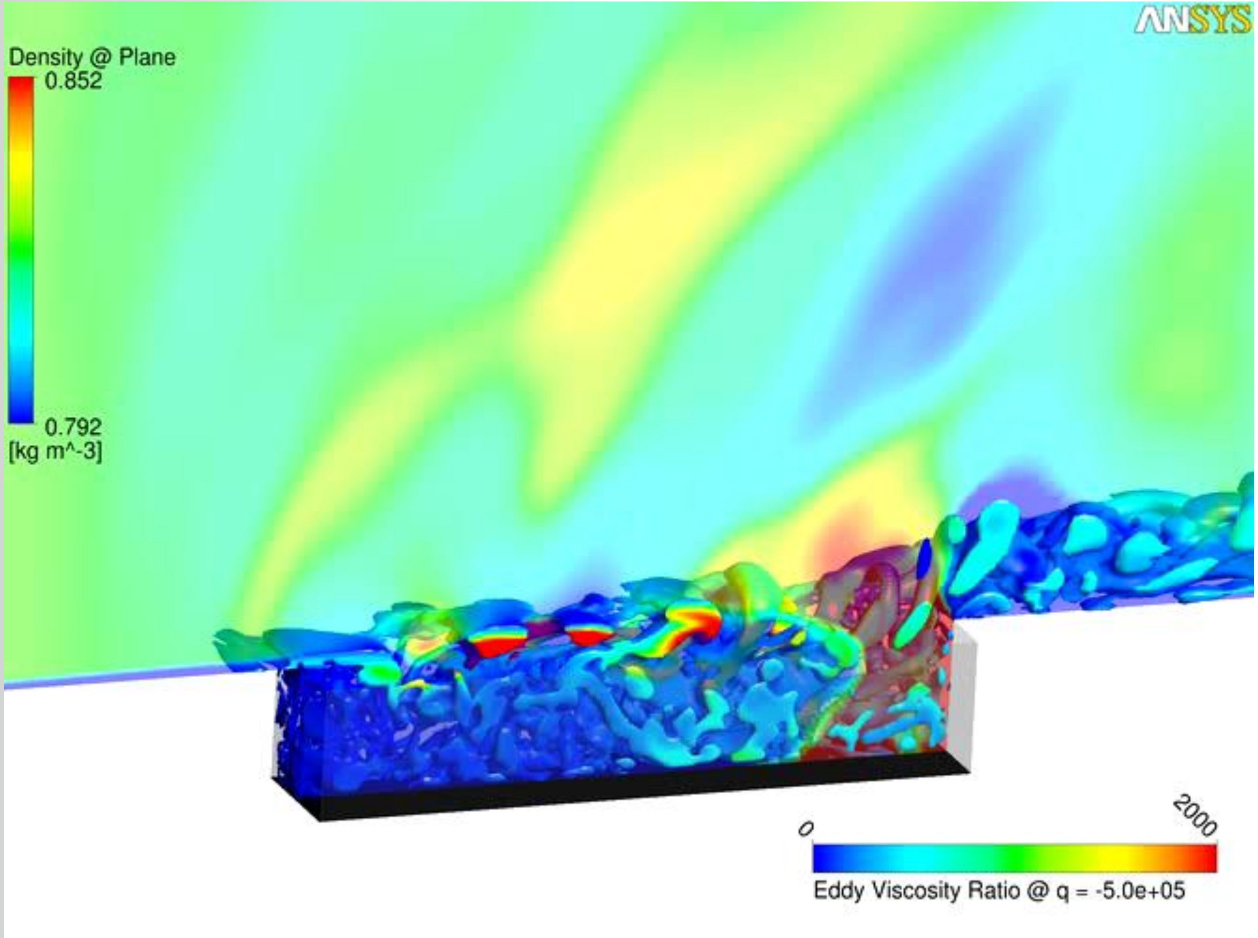
Turbulent structure by q-criterion

Eddy viscosity ratio @ q = -500000 ($q = {}^{1}/_{2}\,(S{:}S - \Omega{:}\Omega)$)

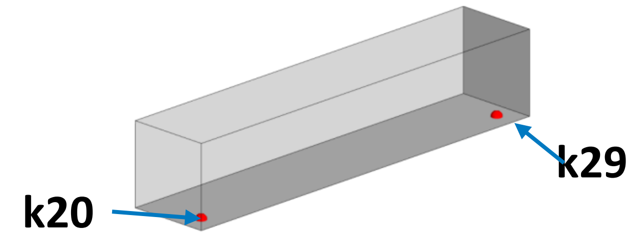


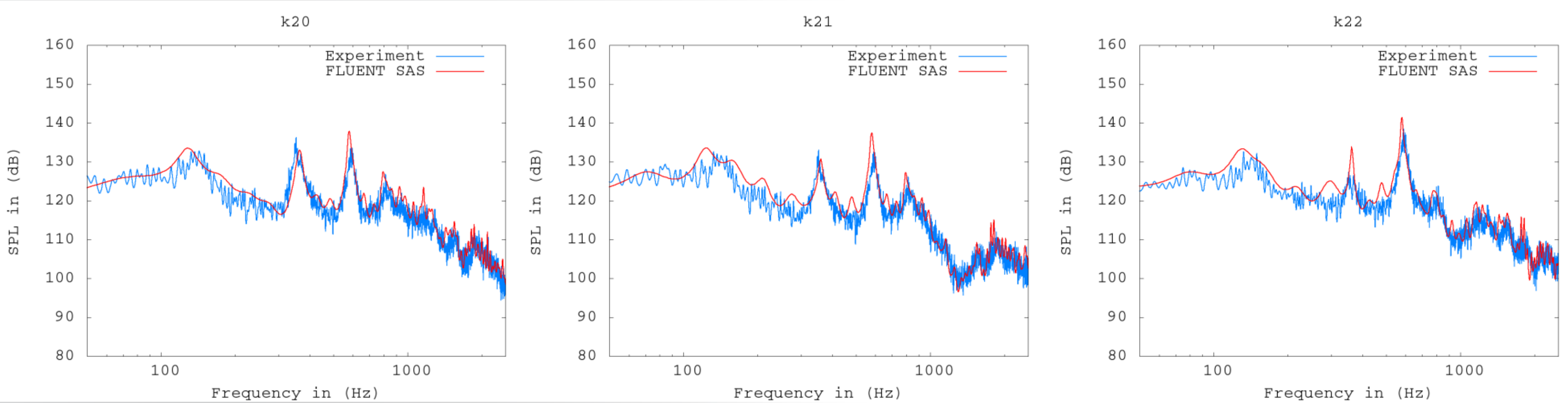
Wave propagation by Fluctuating Density

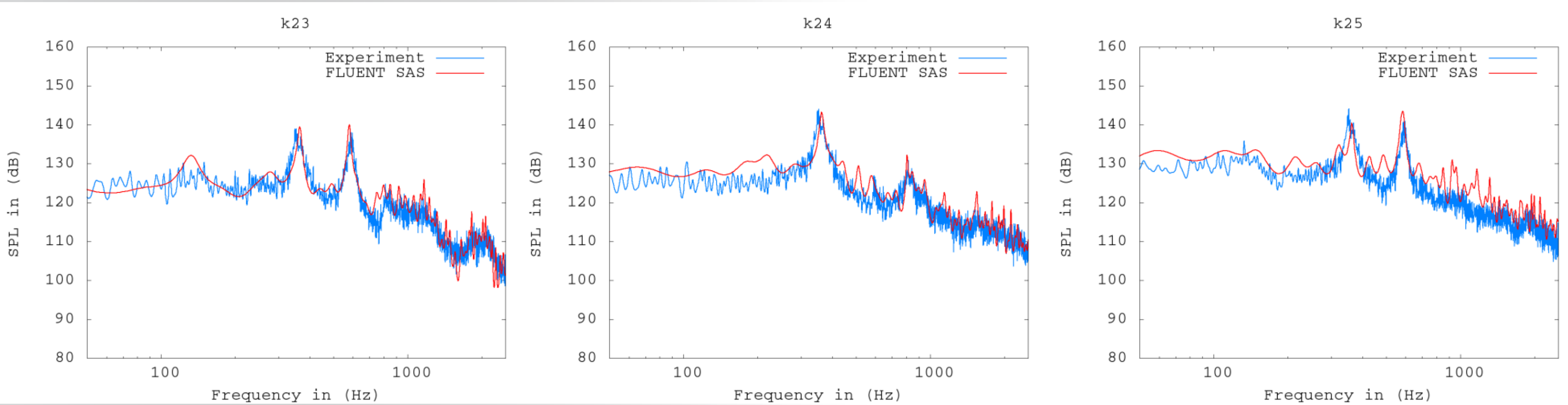
Eddy viscosity ratio @ q = -500000 ($q = {}^1/_2\,(S{:}S - \Omega{:}\Omega)$)



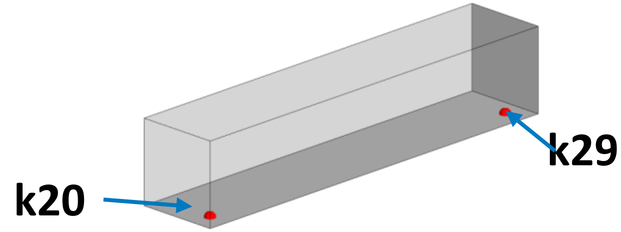
k20 – k25

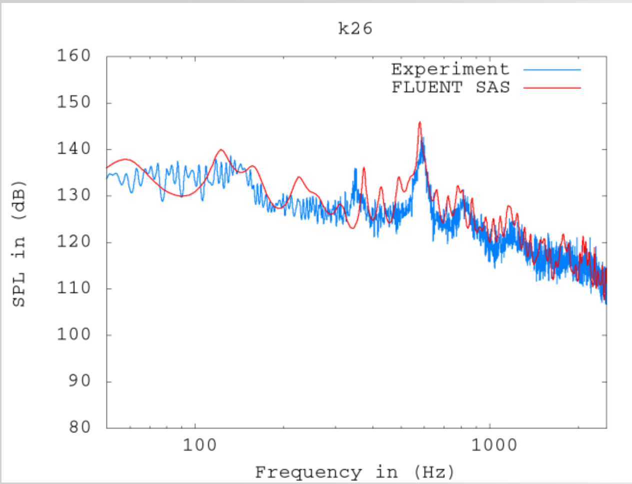

k20
k29

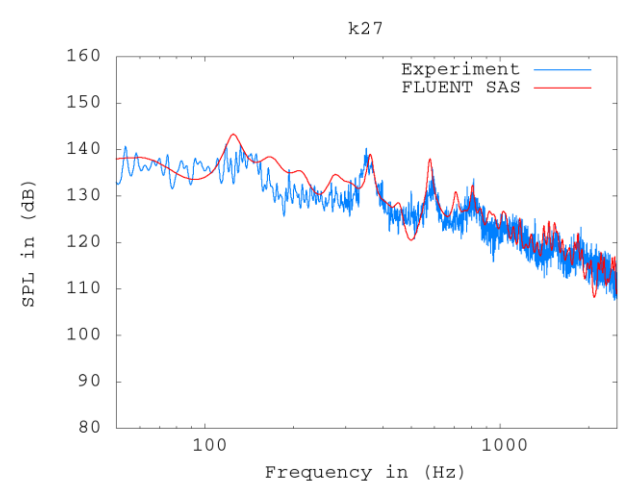

k20
k21
k22
Experiment
FLUENT SAS
SPL in (dB)
Frequency in (Hz)
160
150
140
130
120
110
100
90
80
100
1000

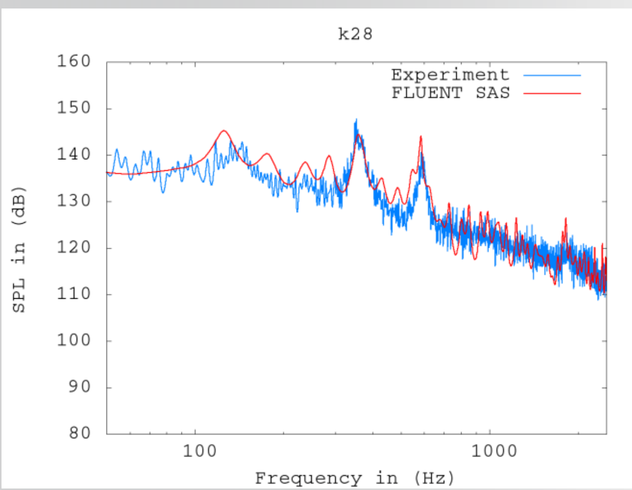

k23
k24
k25
Experiment
FLUENT SAS
SPL in (dB)
Frequency in (Hz)
160
150
140
130
120
110
100
90
80
100
1000

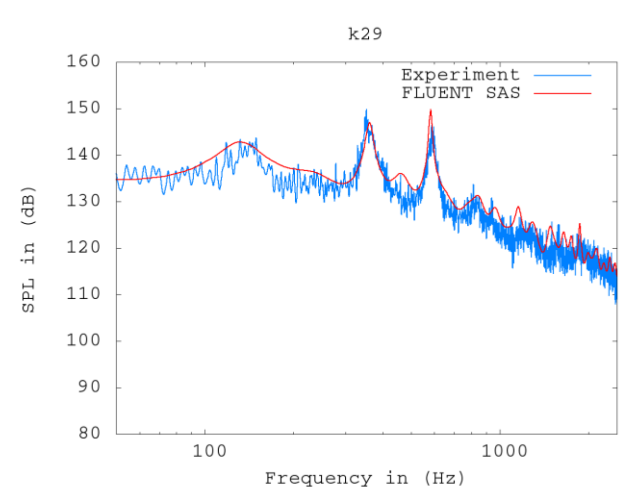
k26 – k29


k20
k29


k26
Experiment
FLUENT SAS
SPL in (dB)
160
150
140
130
120
110
100
90
80
100
1000
Frequency in (Hz)

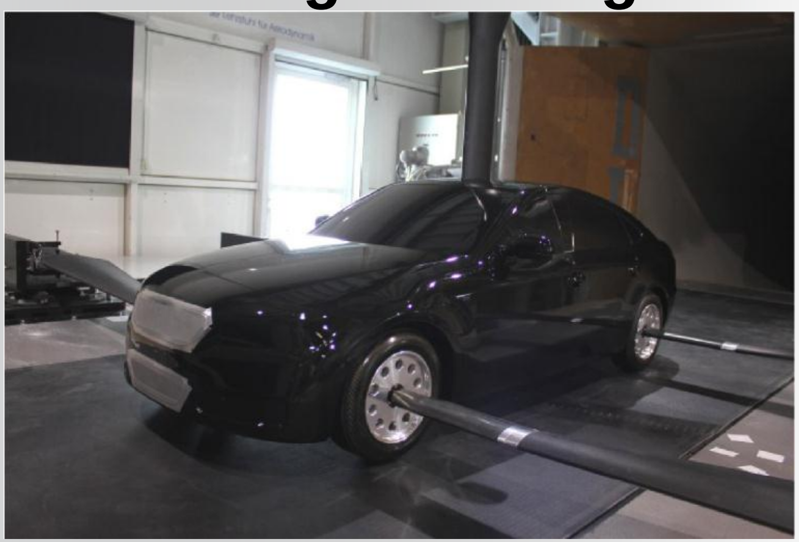

k27
Experiment
FLUENT SAS
SPL in (dB)
160
150
140
130
120
110
100
90
80
100
1000
Frequency in (Hz)


k28
Experiment
FLUENT SAS
SPL in (dB)
160
150
140
130
120
110
100
90
80
100
1000
Frequency in (Hz)

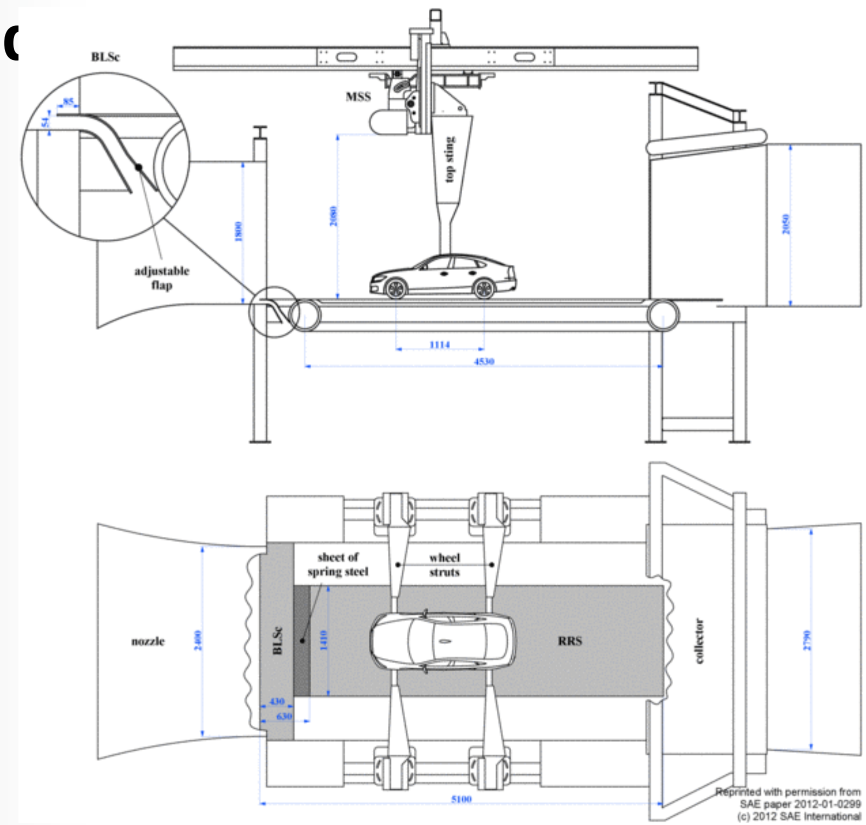

k29
Experiment
FLUENT SAS
SPL in (dB)
160
150
140
130
120
110
100
90
80
100
1000
Frequency in (Hz)

Testcase Description – Experimental Test Facility and Data

- **The experimental data is provided by the Institute of Aerodynamics and Fluid Mechanics from TUM (not yet released)**
- **Experiments are performed including a moving belt**



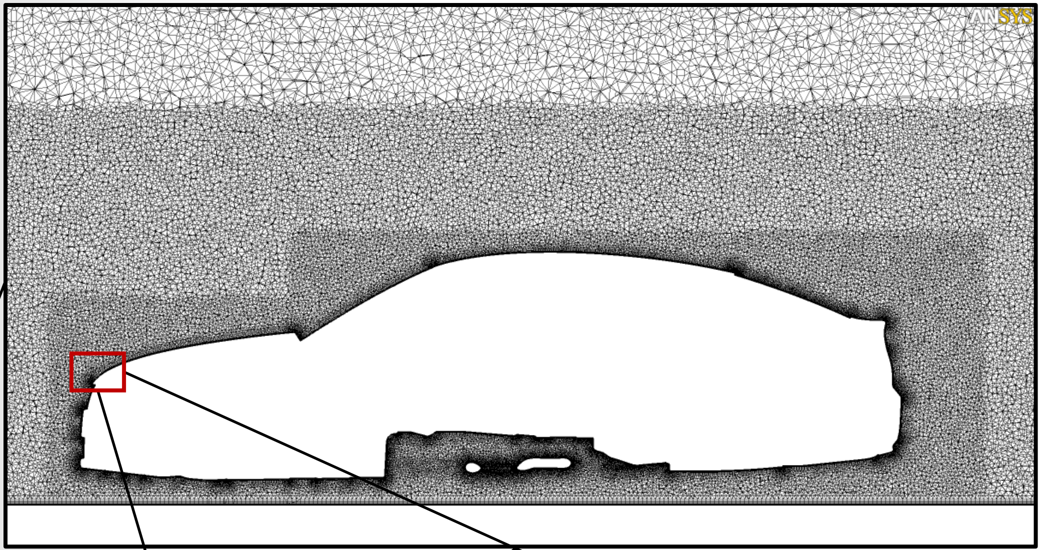
Courtesy by TU Munich, Inst. of Aerodynamics

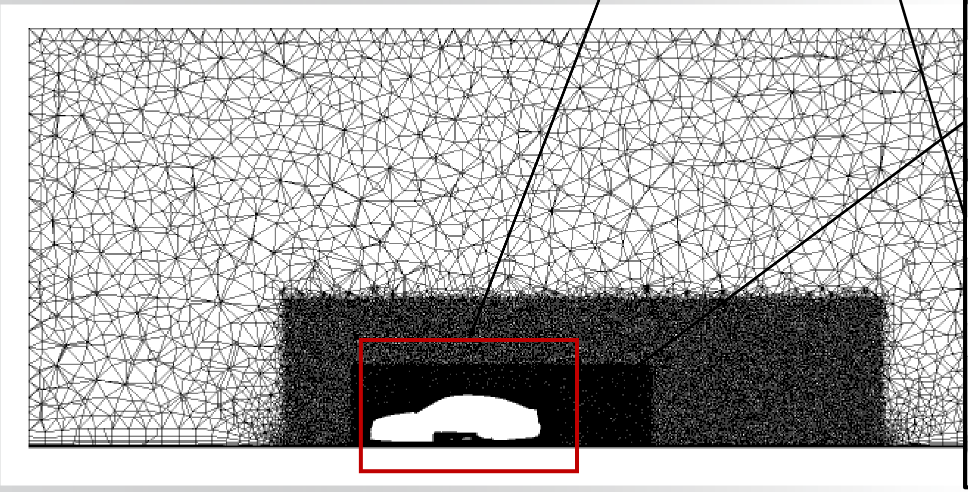


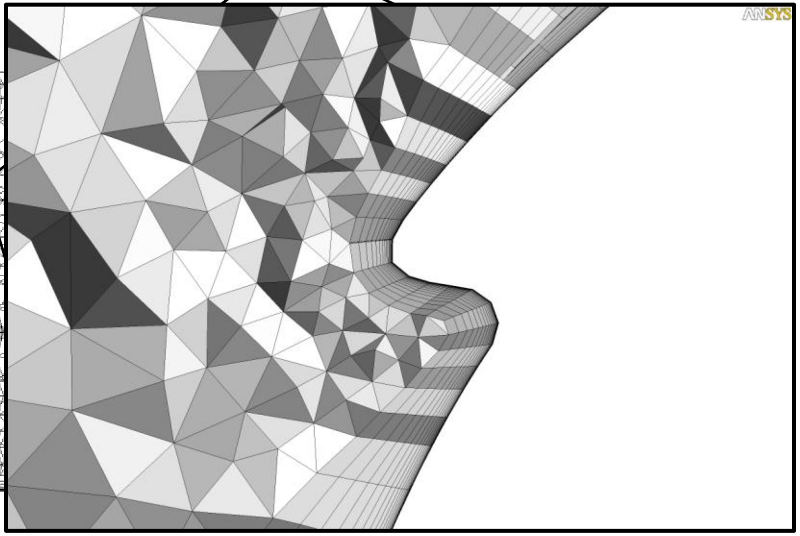
Computational Mesh 2

- **108,034,893 Cells**
- **Four Refinement Boxes**
- **MRF-Zones**

	Number of Inflations	First layer height
Car	20	0.02 mm
Road	20	0.02 mm



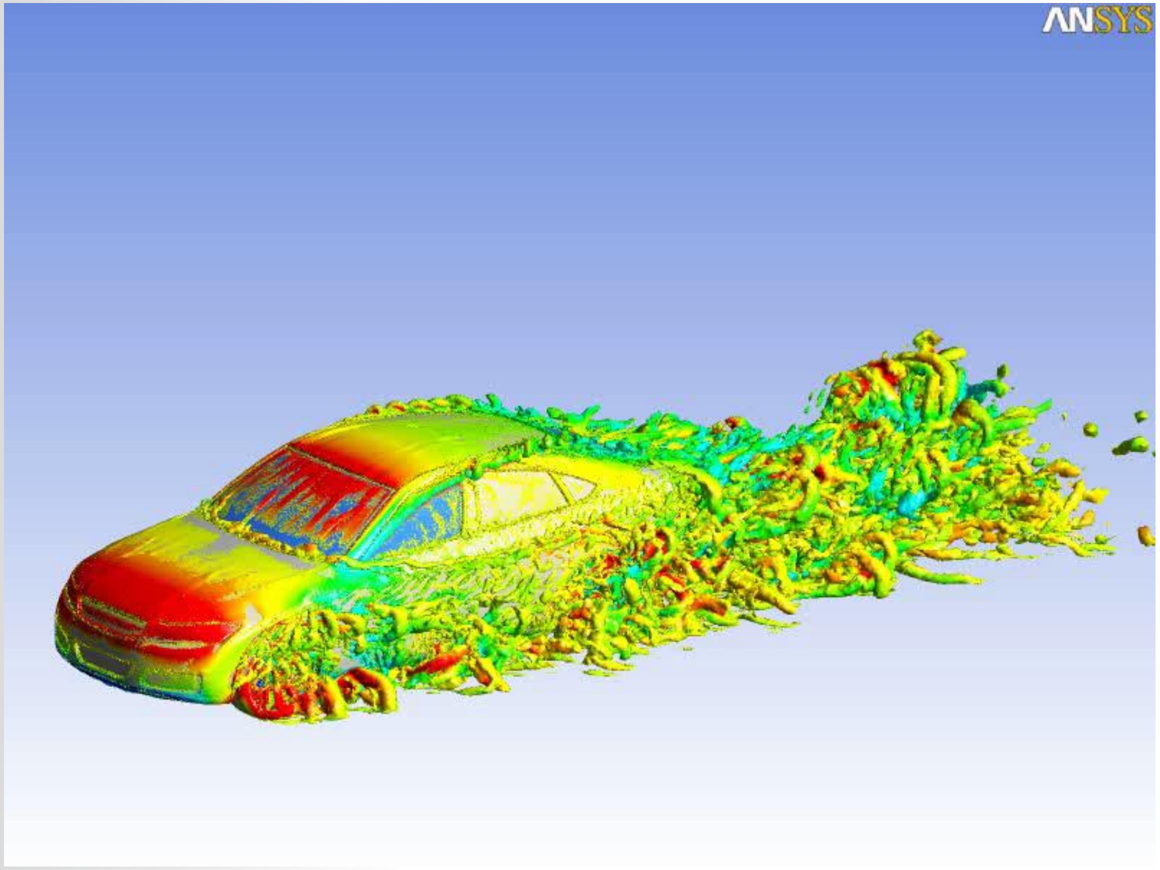




Courtesy by TU Munich, Inst. of Aerodynamics

DrivAir Generic Car Model

- Courtesy Tu Munich
- Currently studied with ANSYS CFD (Fluent and CFX)
- Data not yet public



Overall Summary

- SAS is a second generation URANS model
  - It is derived on URANS arguments
  - It can resolve turbulence structures with LES quality
  - A strong flow instability is required to generate new – resolved turbulence
- Examples
  - Flows past bluff bodies
  - Strongly swirling flows (combustion chamber)
  - Strongly interacting flows (mixing of two jets etc.)
- SAS Model is first and relatively save step into Scale-Resolving Simulations **(**SRS) modeling
  - Worthwhile to try
  - Alternative Detached Eddy Simulation (DES)

